Category Theory
Fall 2022

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Assignment 7
Hand in date: Wed Dec 7

Exercise 1. *Hand in your solution to Exercise* 4.10 *from the notes on categorical logic.*

Exercise 2. *Hand in your solution to Exercise* 4.14 *from the notes on categorical logic.*

Exercise 3. *Hand in your solution to Exercise* 5.3 *from the notes on categorical logic.*

Exercise 4. *Let* $\left(X, \left(\stackrel{i}{=}\right)_{i=0}^{\infty}\right)$ *be a complete ordered family of equivalences,* $\{x_i\}_{i=0}^{\infty}$ *and* $\{y_i\}_{i=0}^{\infty}$ *two Cauchy sequences in* X*, and* $n \in \mathbb{N}$.

Show that if $x_i \stackrel{n}{=} y_i$ *for all* $i \in \mathbb{N}$ *then*

$$\lim_{i\to\infty} x_i \stackrel{n}{=} \lim_{i\to\infty} y_i.$$

Exercise 5. *Let* X *and* Y *be two complete ordered families of equivalences and let* X *be inhabited. Let* $f : Y \times X \to X$ *be a non-expansive function such that for all* $y \in Y$ *the function* $f(y,-) : X \to X$ *is* contractive.

Show that there exists a unique non-expansive *function* $g : Y \to X$ *such that* $f(y, g(y)) = g(y)$.